MATLAB Examples 2 (covering Statistics Lectures 3 and 4)

Contents

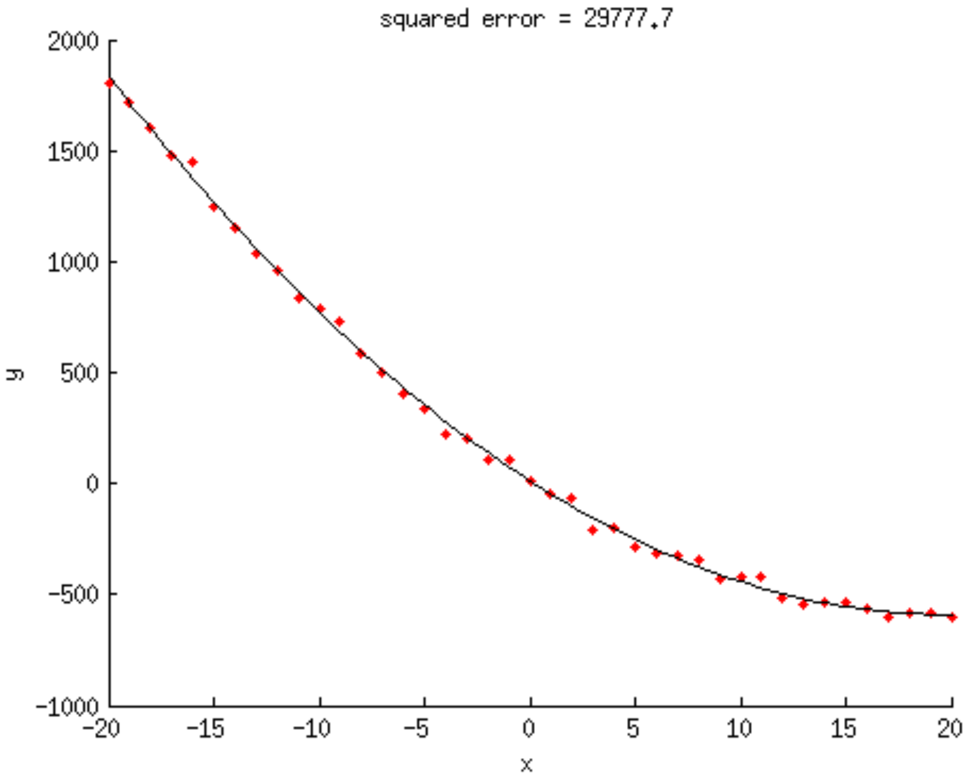
Example 1: Fit a linearized regression model

```
% generate some random data
x = -20:20;
y = 1.5*x.^2 - 60*x + 10 + 30*randn(size(x));

% given these data, we will now try to fit a model
% to the data.  we assume we know what model to use,
% namely, y = ax^2 + bx + c where a, b, and c are
% free parameters.  notice that this was the model
% we used to generate the data.

% since our model is a linearized model, we can express
% it using simple matrix notation.

% construct the regressor matrix
X = [x(:).^2 x(:) ones(length(x),1)];

% estimate the free parameters of the model
% using ordinary least-squares (OLS) estimation
w = inv(X'*X)*X'*y(:);

% what are the estimated weights?
w
```

```
w =

            1.5368275257135
          -60.638944801903
           8.15661270608602
```

```
% what is the model fit?
modelfit = X*w;

% what is the squared error achieved by this model fit?
squarederror = sum((y(:)-modelfit).^2);

% visualize
figure;
hold on;
scatter(x,y,'r.');
ax = axis;
```

```
xx = linspace(ax(1),ax(2),100)';
yy = [xx.^2 xx ones(length(xx),1)]*w;
plot(xx,yy,'k-');
xlabel('x');
ylabel('y');
title(sprintf('squared error = %.1f',squarederror));
```



Example 2: Fit a parametric nonlinear model

```
% generate some random data
x = 0:.05:3;
y = 5*x.^3.5 + 5*randn(size(x));

% given these data, we will now try to fit a model
% to the data.  we assume we know what model to use,
% namely, y = ax^n where a and n are free parameters.
% notice that this was the model we used to generate the data.

% define optimization options
options = optimset('Display','iter','FunValCheck','on', ...
                   'MaxFunEvals',Inf,'MaxIter',Inf, ...
                   'TolFun',1e-6,'TolX',1e-6);

% define bounds for the parameters
%               a    n
paramslb = [-Inf    0];  % lower bound
paramsub = [ Inf  Inf];  % upper bound

% define the initial seed
%             a      n
```

```
params0 = [  1     1];

% estimate the free parameters of the model using nonlinear optimization
  % hint: pp is a vector of parameters like [a n],
  %       xx is a vector of x-values like [.1 .4 .5],
  %       and what modelfun does is to output the predicted y-values.
modelfun = @(pp,xx) pp(1)*xx.^pp(2);
[params,resnorm,residual,exitflag,output] = lsqcurvefit(modelfun,params0,x,y,paramslb,paramsub
,options);
```

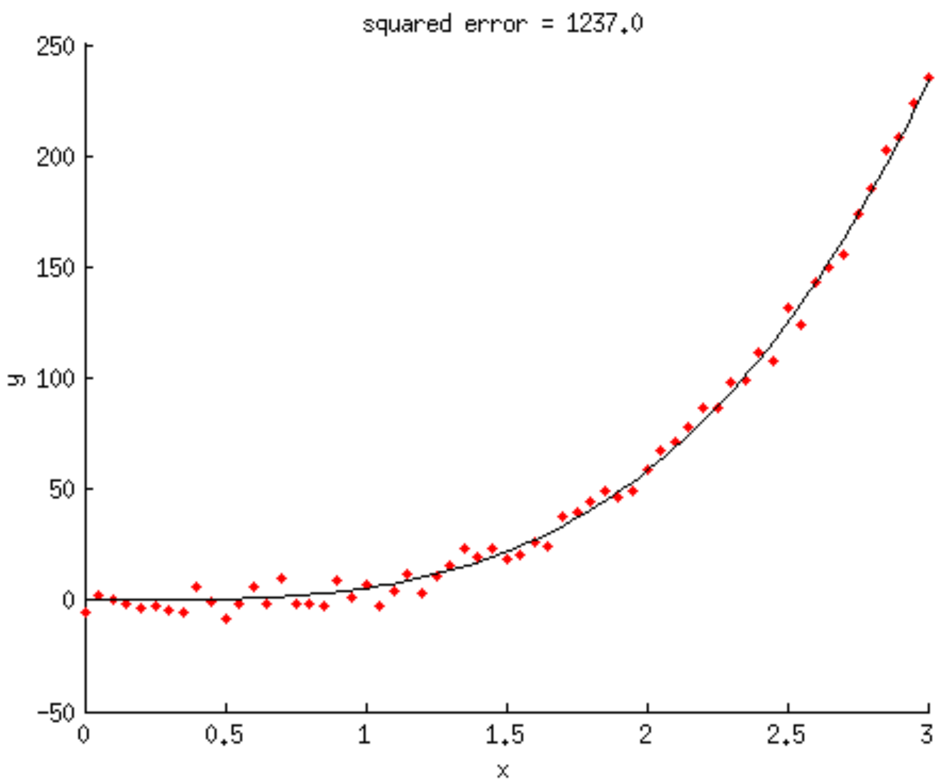
```
                                            Norm of      First-order
 Iteration  Func-count      f(x)           step          optimality   CG-iterations
     0           3         431921                         7.93e+03
     1           6         431921               10        7.93e+03              0
     2           9         247013              2.5         8.3e+04              0
     3          12         247013          2.81299         8.3e+04              0
     4          15        98286.2         0.703246        1.09e+05              0
     5          18        17315.7          1.40649        3.78e+05              0
     6          21        1894.05         0.327179        2.46e+04              0
     7          24        1482.46         0.939262        9.09e+03              0
     8          27        1241.08         0.422605        1.33e+03              0
     9          30        1237.01       0.00758631             5.5              0
    10          33        1237.01      1.29563e-05        7.82e-05              0

Local minimum possible.

lsqcurvefit stopped because the final change in the sum of squares relative to
its initial value is less than the selected value of the function tolerance.
```

```
% what are the estimated parameters?
params
```

```
params =

           5.4338041940236          3.42528833322854
```

```
% what is the model fit?
modelfit = modelfun(params,x);

% what is the squared error achieved by this model fit?
squarederror = sum((y(:)-modelfit(:)).^2);
% (note that the 'resnorm' output also provides the squared error.)

% visualize
figure;
hold on;
scatter(x,y,'r.');
```

```
ax = axis;
xx = linspace(ax(1),ax(2),100)';
yy = modelfun(params,xx);
plot(xx,yy,'k-');
xlabel('x');
ylabel('y');
title(sprintf('squared error = %.1f',squarederror));
```



Example 3: Another optimization example

```
% generate some random data
x = rand(1,1000).^2;

% we want to determine the value that minimizes the sum of the
% absolute differences between the value and the data points

% define some stuff
options = optimset('Display','iter','FunValCheck','on', ...
                   'MaxFunEvals',Inf,'MaxIter',Inf, ...
                   'TolFun',1e-6,'TolX',1e-6);
paramslb = [];  % [] means no bounds
paramsub = [];
params0 = [0];

% perform the optimization
  % hint: here we have to apply a square-root transformation so that when
  % lsqnonlin squares the output of costfun, we will be back to absolute error.
costfun = @(pp) sqrt(abs(x-pp));
[params,resnorm,residual,exitflag,output] = lsqnonlin(costfun,params0,paramslb,paramsub,option
s);
```

```
                                         Norm of      First-order
 Iteration  Func-count     f(x)          step          optimality   CG-iterations
     0          2          346.47                          500
     1          4         346.385     8.61928e-05          490             0
     2          6         341.862      0.00491093          440             0
     3          8         333.538       0.0101418          387             0
     4         10         315.811       0.0259447          306             0
     5         12         308.364       0.0129775          271             0
     6         14         292.776       0.0324211          215             0
     7         16          279.22        0.035143          170             0
     8         18          270.08       0.0314543          118             0
     9         20         263.724        0.031138           88             0
    10         22         259.555       0.0265819           67             0
    11         24         258.266       0.0107348           52             0
    12         26         257.181        0.012298           35             0
    13         28         256.649      0.00864949           25             0
    14         30         256.518      0.00310659           19             0
    15         32         256.401      0.00335775           15             0
    16         34         256.317      0.00312291           12             0
    17         36         256.252      0.00307883            7             0
    18         38         256.245     0.000546973            6             0
    19         40         256.236     0.000864426            5             0
    20         42         256.227       0.0010377            4             0
    21         44         256.224     0.000510969            3             0
    22         46          256.22     0.000677599            3             0
    23         48         256.218     0.000404828            1             0
    24         50         256.218     0.000140253            1             0
    25         52         256.217     0.000193302            1             0
    26         54         256.217      0.00021431            1             0
    27         56         256.217     0.000205629            1             0
    28         58         256.216     9.93736e-05     5.88e-05             0

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to
its initial value is less than the selected value of the function tolerance.
```

```
% what is the solution?
params
```

```
params =

         0.260043242623885
```

```
% what is the median of the data?
median(x)
```

```
ans =

        0.260365029783367
```

```
% notice that the solution is nearly identical to the median of the data.
% (the slop comes from the tolerance used in the optimization and
% from the interpolation used in the calculation of the median.)
```

Published with MATLAB® R2012b