

## MATLAB Examples 4 (covering Statistics Lecture 7)

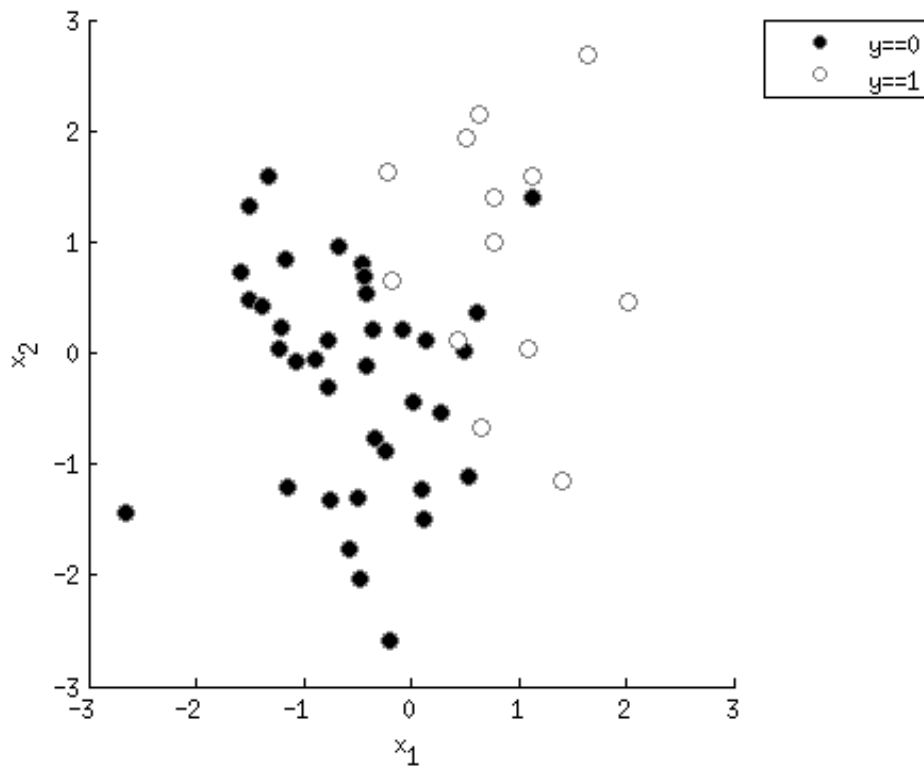
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### Example 1: Simple 2D classification using logistic regression

```
% generate some data (50 data points defined in two dimensions;
% class assignment is 0 or 1 for each data point)
x1 = randn(50,1);
x2 = randn(50,1);
y = (2*x1 + x2 + randn(size(x1))) > 1;

% visualize the data
figure; hold on;
h1 = scatter(x1(y==0),x2(y==0),50,'k','filled'); % black dots for 0
h2 = scatter(x1(y==1),x2(y==1),50,'w','filled'); % white dots for 1
set([h1 h2],'MarkerEdgeColor',[.5 .5 .5]); % outline dots in gray
legend([h1 h2],{'y==0' 'y==1'},'Location','NorthEastOutside');
xlabel('x_1');
ylabel('x_2');
```



```
% define optimization options
options = optimset('Display','iter','FunValCheck','on', ...
                  'MaxFunEvals',Inf,'MaxIter',Inf, ...
```

```

        'TolFun',1e-6,'TolX',1e-6);

% define logistic function (this takes a matrix of values and
% applies the logistic function to each value)
logisticfun = @(x) 1./(1+exp(-x));

% construct regressor matrix
X = [x1 x2];
X(:,end+1) = 1; % we need to add a constant regressor

% define initial seed
params0 = zeros(1,size(X,2));

% define bounds
paramslb = []; % [] means no bounds
paramsub = [];

% define cost function (negative-log-likelihood)
% here, the eps is a hack to ensure that the log returns a finite number.
costfun = @(pp) -sum((y .* log(logisticfun(X*pp')+eps)) + ...
                    ((1-y) .* log(1-logisticfun(X*pp')+eps))));

% fit the logistic regression model by finding
% parameter values that minimize the cost function
[params,resnorm,residual,exitflag,output] = ...
    lsqnonlin(costfun,params0,paramslb,paramsub,options);

```

Warning: Trust-region-reflective algorithm requires at least as many equations as variables; using Levenberg-Marquardt algorithm instead.

Iteration	Func-count	Residual	First-Order optimality	Lambda	Norm of step
0	4	1201.13	544	0.01	
1	8	249.865	66.7	0.001	1.57413
2	12	162.838	16.1	0.0001	3.09029
3	22	157.936	10.8	100	0.168952
4	26	155.321	7.6	10	0.120881
5	30	153.054	14.6	1	0.91166
6	36	148.397	7.02	100	0.173822
7	40	147.149	3.24	10	0.090581
8	45	146.815	1.57	100	0.046862
9	49	146.718	1.02	10	0.0248996
10	54	146.682	0.807	100	0.01455
11	58	146.665	0.669	10	0.00997112
12	62	146.643	0.809	1	0.0786498
13	68	146.624	0.377	100	0.0113596
14	72	146.619	0.206	10	0.00539815
15	77	146.618	0.12	100	0.00271585
16	81	146.618	0.0742	10	0.00154244
17	86	146.618	0.062	100	0.00104312
18	90	146.617	0.0539	10	0.000814137

Local minimum possible.

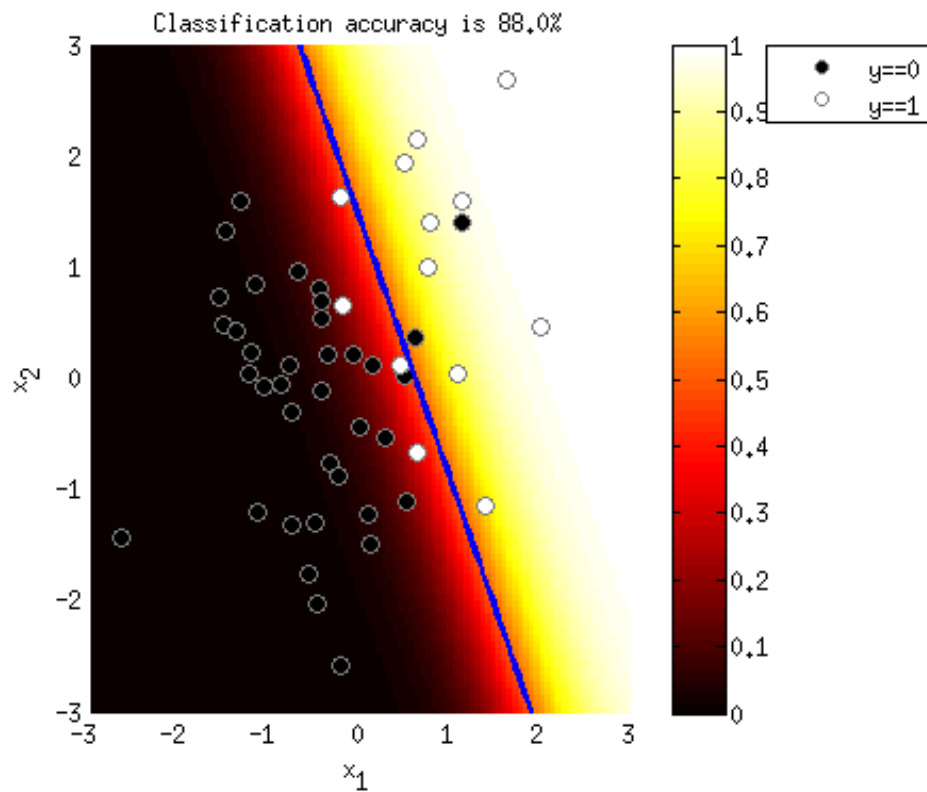
lsqnonlin stopped because the final change in the sum of squares relative to

its initial value is less than the selected value of the function tolerance.

```
% compute the output (class assignment) of the model for each data point
modelfit = logisticfun(X*params') >= 0.5;

% calculate percent correct (percentage of data points
% that are correctly classified by the model)
pctcorrect = sum(modelfit==y) / length(y) * 100;

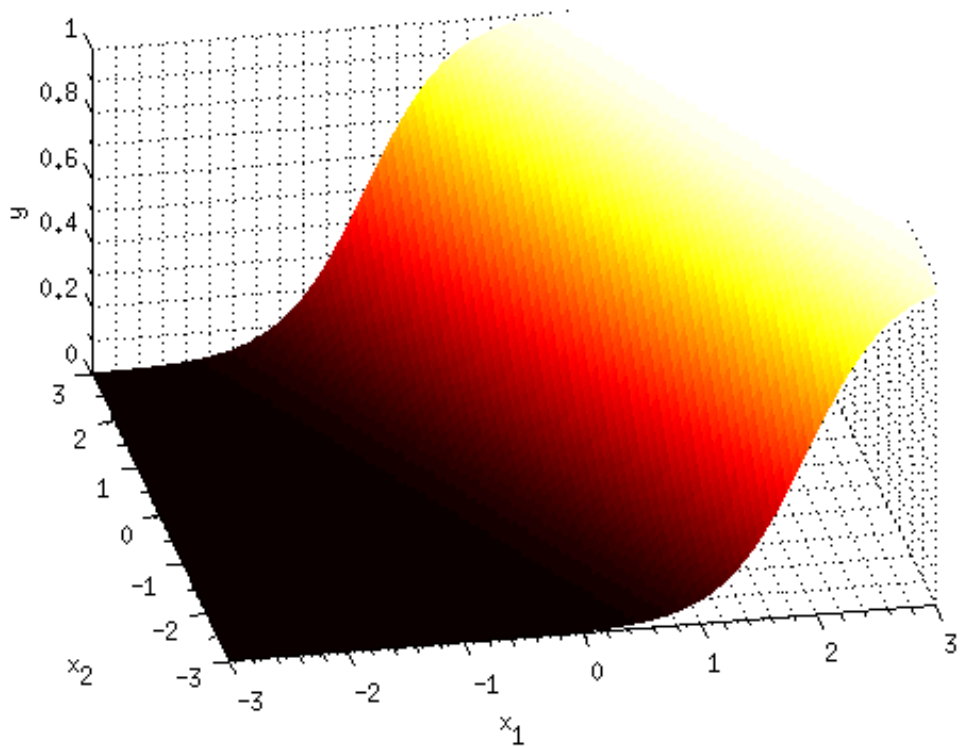
% visualize the model
% prepare a grid of points to evaluate the model at
ax = axis;
xvals = linspace(ax(1),ax(2),100);
yvals = linspace(ax(3),ax(4),100);
[xx,yy] = meshgrid(xvals,yvals);
% construct regressor matrix
X = [xx(:) yy(:)];
X(:,end+1) = 1;
% evaluate model at the points (but don't perform the final thresholding)
outputimage = reshape(logisticfun(X*params'),[length(yvals) length(xvals)]);
% visualize the image (the default coordinate system for images
% is 1:N where N is the number of pixels along each dimension.
% we have to move the image to the proper position; we
% accomplish this by setting XData and YData.)
h3 = imagesc(outputimage,[0 1]); % the range of the logistic function is 0 to 1
set(h3,'XData',xvals,'YData',yvals);
colormap(hot);
colorbar;
% visualize the decision boundary associated with the model
% by computing the 0.5-contour of the image
[c4,h4] = contour(xvals,yvals,outputimage,[.5 .5]);
set(h4,'LineWidth',3,'LineColor',[0 0 1]); % make the line thick and blue
% send the image to the bottom so that we can see the data points
uistack(h3,'bottom');
% send the contour to the top
uistack(h4,'top');
% restore the original axis range
axis(ax);
% report the accuracy of the model in the title
title(sprintf('Classification accuracy is %.1f%%',pctcorrect));
```



```

% try a 3D visualization
figure; hold on;
h1 = surf(xvals,yvals,outputimage);
set(h1,'EdgeAlpha',0); % don't show the edges of the surface
colormap(hot);
xlabel('x_1');
ylabel('x_2');
zlabel('y');
view(-11,42); % set the viewing angle
% show some grid lines for reference
set(gca,'XGrid','on','XMinorGrid','on', ...
      'YGrid','on','YMinorGrid','on', ...
      'ZGrid','on','ZMinorGrid','on');

```



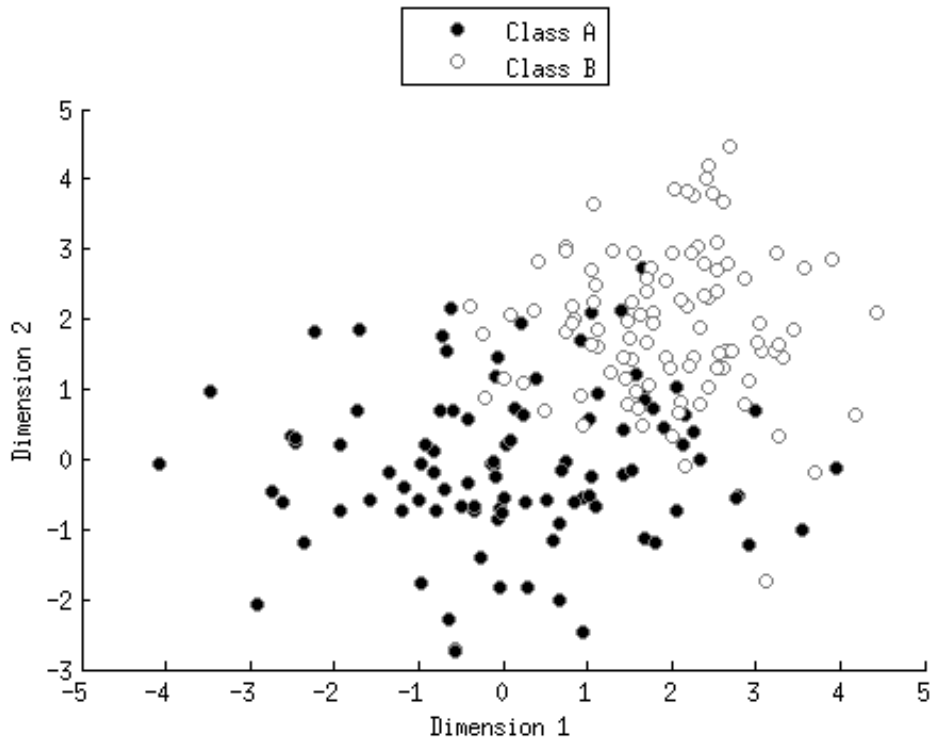
## Example 2: Compare solutions of different classifiers

```

% generate some data (two classes of data points defined in two dimensions)
classA = bsxfun(@times,[1.5 1]',randn(2,100));
classB = 2+randn(2,100);

% visualize the data
figure; hold on;
h1 = scatter(classA(1,:),classA(2:,:), 'ko', 'filled');
h2 = scatter(classB(1,:),classB(2:,:), 'wo', 'filled');
set([h1 h2], 'MarkerEdgeColor', [.5 .5 .5]);
legend([h1 h2], {'Class A' 'Class B'}, 'Location', 'NorthOutside');
xlabel('Dimension 1');
ylabel('Dimension 2');

```



```

% prepare a grid of points to evaluate the model at
ax = axis;
xvals = linspace(ax(1),ax(2),200);
yvals = linspace(ax(3),ax(4),200);
[xx,yy] = meshgrid(xvals,yvals);
gridX = [xx(:) yy(:)];

% perform logistic regression (here we use the MATLAB function glmfit.m
% instead of the direct implementation shown in Example 1)
X = [classA(1,:) classA(2,:);
     classB(1,:) classB(2,:)];
y = [zeros(size(classA,2),1); ones(size(classB,2),1)];
paramsA = glmfit(X,y,'binomial','link','logit');
outputimageA = glmval(paramsA,gridX,'logit');
outputimageA = reshape(outputimageA,[length(yvals) length(xvals)]);
[d,hA] = contour(xvals,yvals,outputimageA,[.5 .5]);
set(hA,'LineWidth',3,'LineColor',[0 0 1]); % make the line thick and blue

% perform linear discriminant analysis (here we use the MATLAB function
% classify.m; ideally, we would implement it from scratch so we really
% understand how it works!)
outputimageB = classify(gridX,X,y,'linear');
outputimageB = reshape(outputimageB,[length(yvals) length(xvals)]);
[d,hB] = contour(xvals,yvals,outputimageB,[.5 .5]);
set(hB,'LineWidth',3,'LineColor',[0 1 0]); % make the line thick and green

% nearest-prototype classification
centroidA = mean(classA,2); % 2 x 1
centroidB = mean(classB,2); % 2 x 1
% compute distance to centroid A
disttoA = sqrt(sum(bsxfun(@minus,gridX,centroidA).^2,2));

```

```

% compute distance to centroid B
disttoB = sqrt(sum(bsxfun(@minus,gridX,centroidB').^2,2));
outputimageC = disttoA > disttoB;
outputimageC = reshape(outputimageC,[length(yvals) length(xvals)]);
[d,hC] = contour(xvals,yvals,outputimageC,[.5 .5]);
set(hC,'LineWidth',3,'LineColor',[1 0 0]); % make the line thick and red

% add legend
legend([hA hB hC],{'logistic regression' ...
                  'linear discriminant analysis (LDA)' ...
                  'nearest-prototype classification'}, ...
      'Location','NorthOutside');

```

