# Statistics and Data Analysis in MATLAB

# Lecture 5:
# Model accuracy

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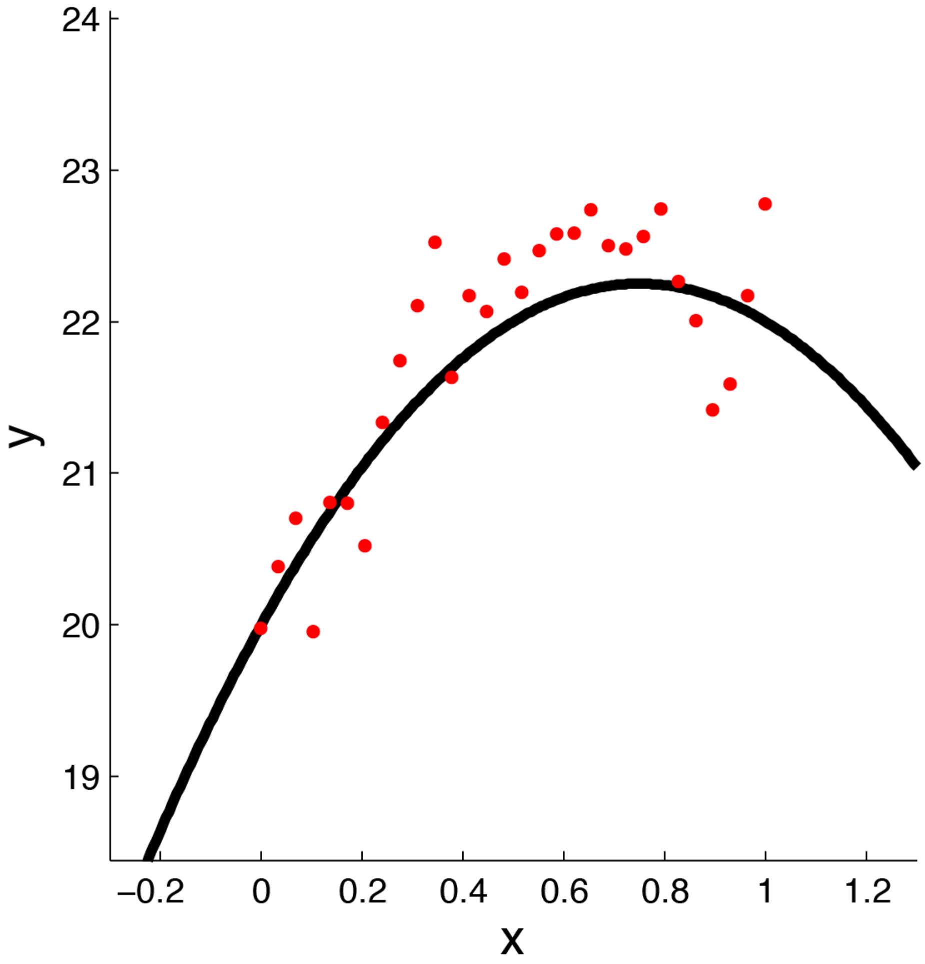
# Quantifying model accuracy



Squared error = 5.4
(dependent on units, hard to interpret)

$R^2$ = 75%
(independent of units, easy to interpret)

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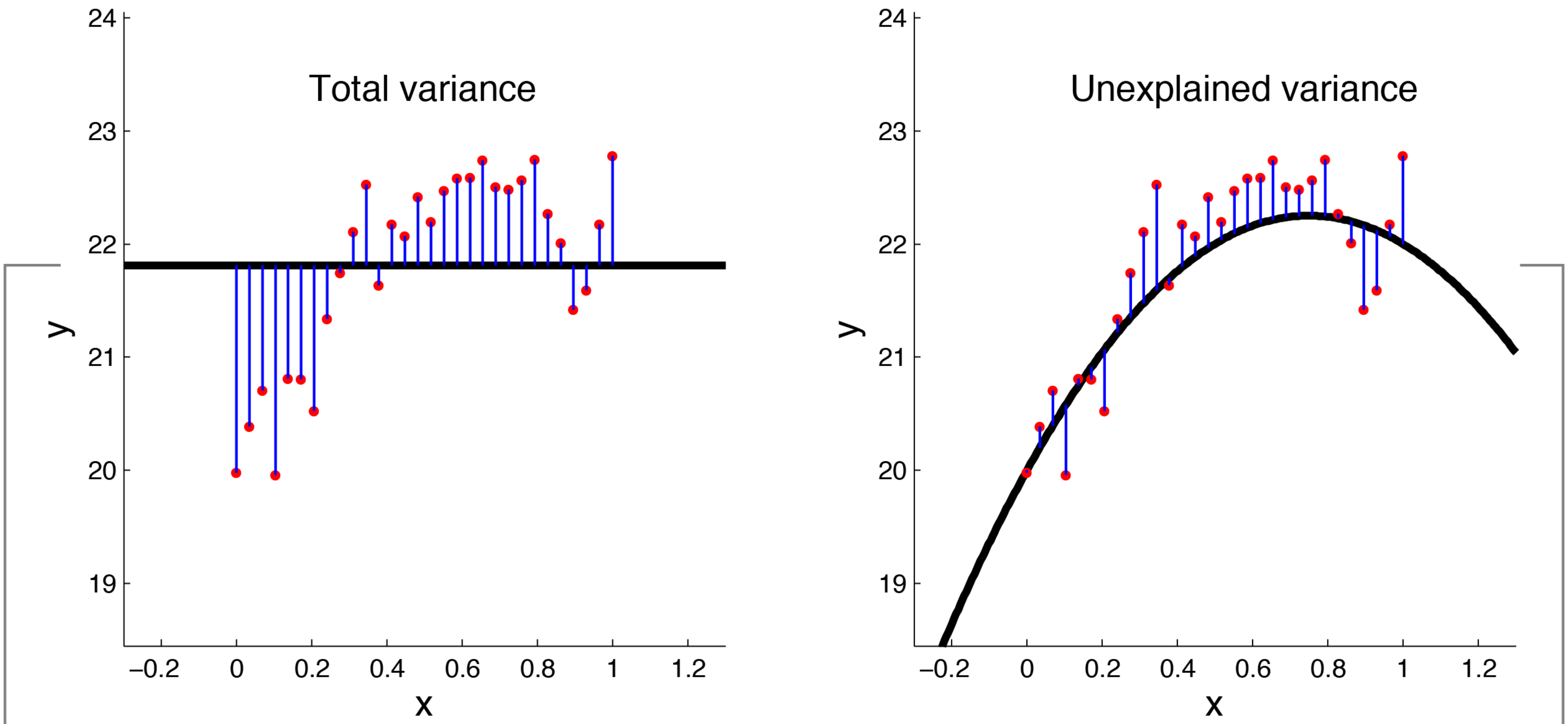
# Variance

$$\text{variance} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

# Coefficient of determination ($R^2$)



$$R^2 = 100 \times \left( 1 - \frac{\text{unexplained variance}}{\text{total variance}} \right)$$

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# Coefficient of determination ($R^2$)

$$R^2 = \text{percent explained variance}$$

$$R^2 = 100 \times \left(\text{fraction explained variance}\right)$$

$$R^2 = 100 \times \left(1 - \frac{\text{unexplained variance}}{\text{total variance}}\right)$$

$$R^2 = 100 \times \left(1 - \frac{\dfrac{\sum_{i=1}^{n}(d_i - m_i)^2}{n-1}}{\dfrac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}}\right)$$

$$R^2 = 100 \times \left(1 - \frac{\sum_{i=1}^{n}(d_i - m_i)^2}{\sum_{i=1}^{n}(d_i - \bar{d})^2}\right)$$

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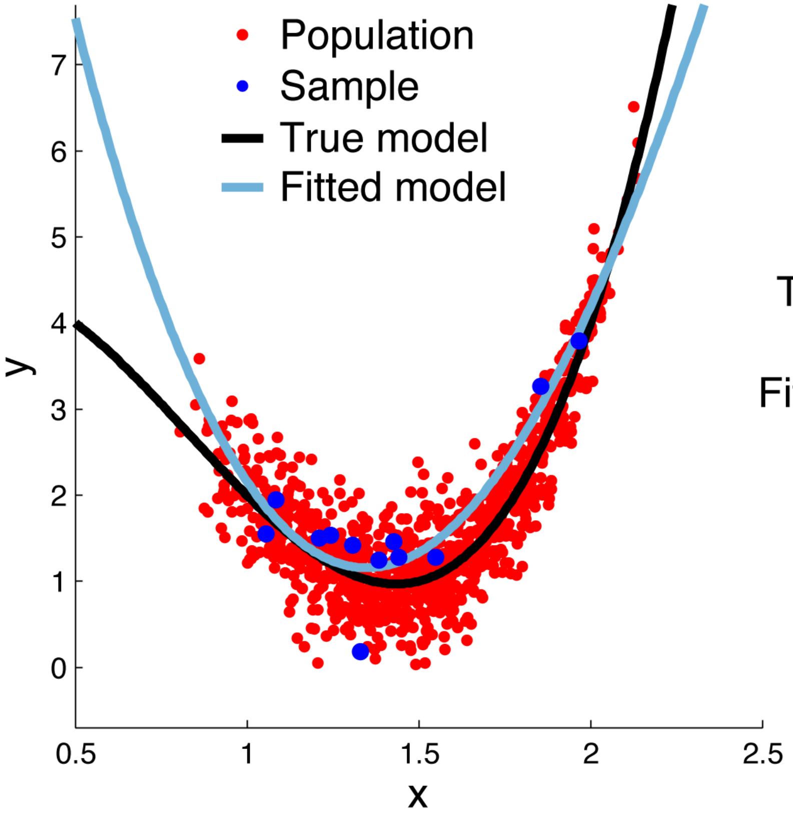
# Direct calculation of *$R^2$* overestimates model accuracy



| | Population | Sample |
|---|---|---|
| True model | high (80%) | high (79%) |
| Fitted model | low (69%) | very high (85%) |

Accuracy of fitted model on sample overestimates true accuracy of fitted model

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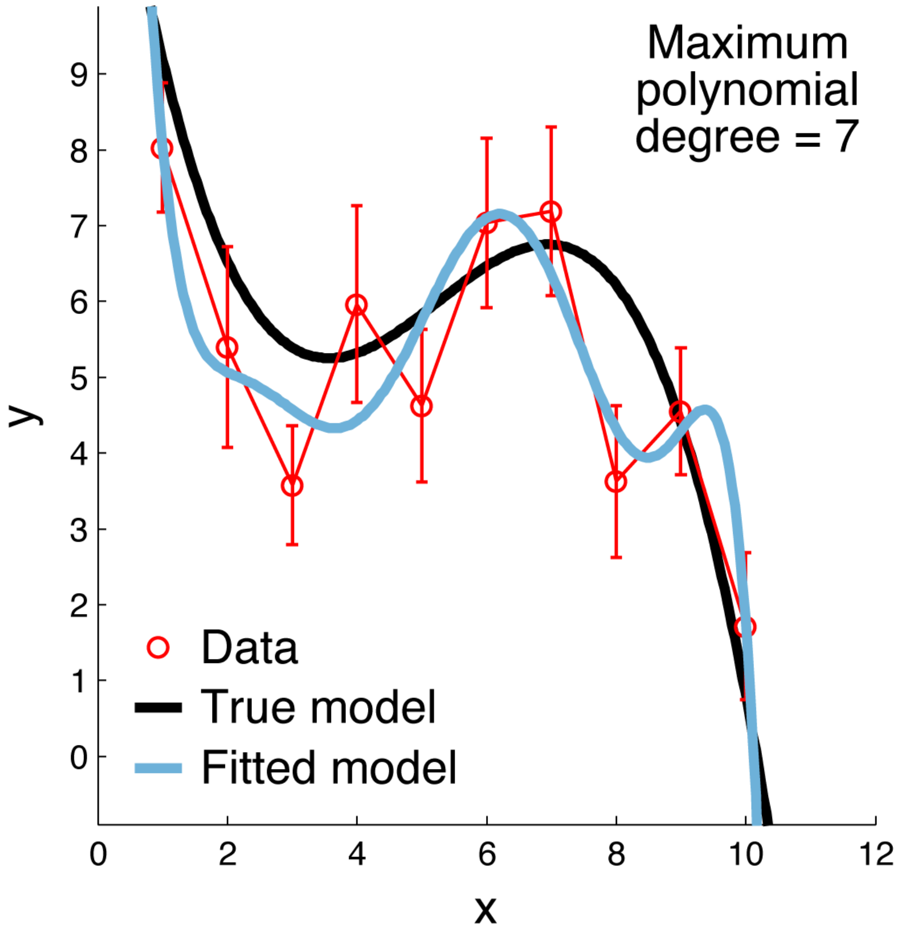
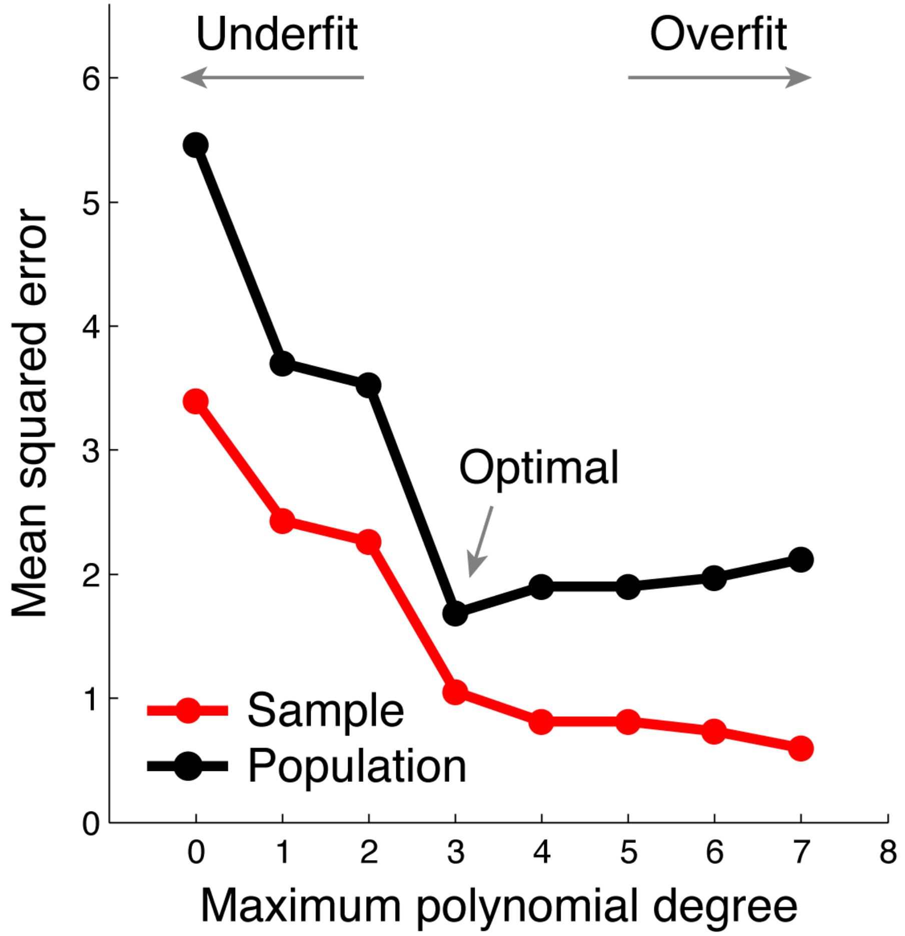
# Overfitting

$$y = ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h$$





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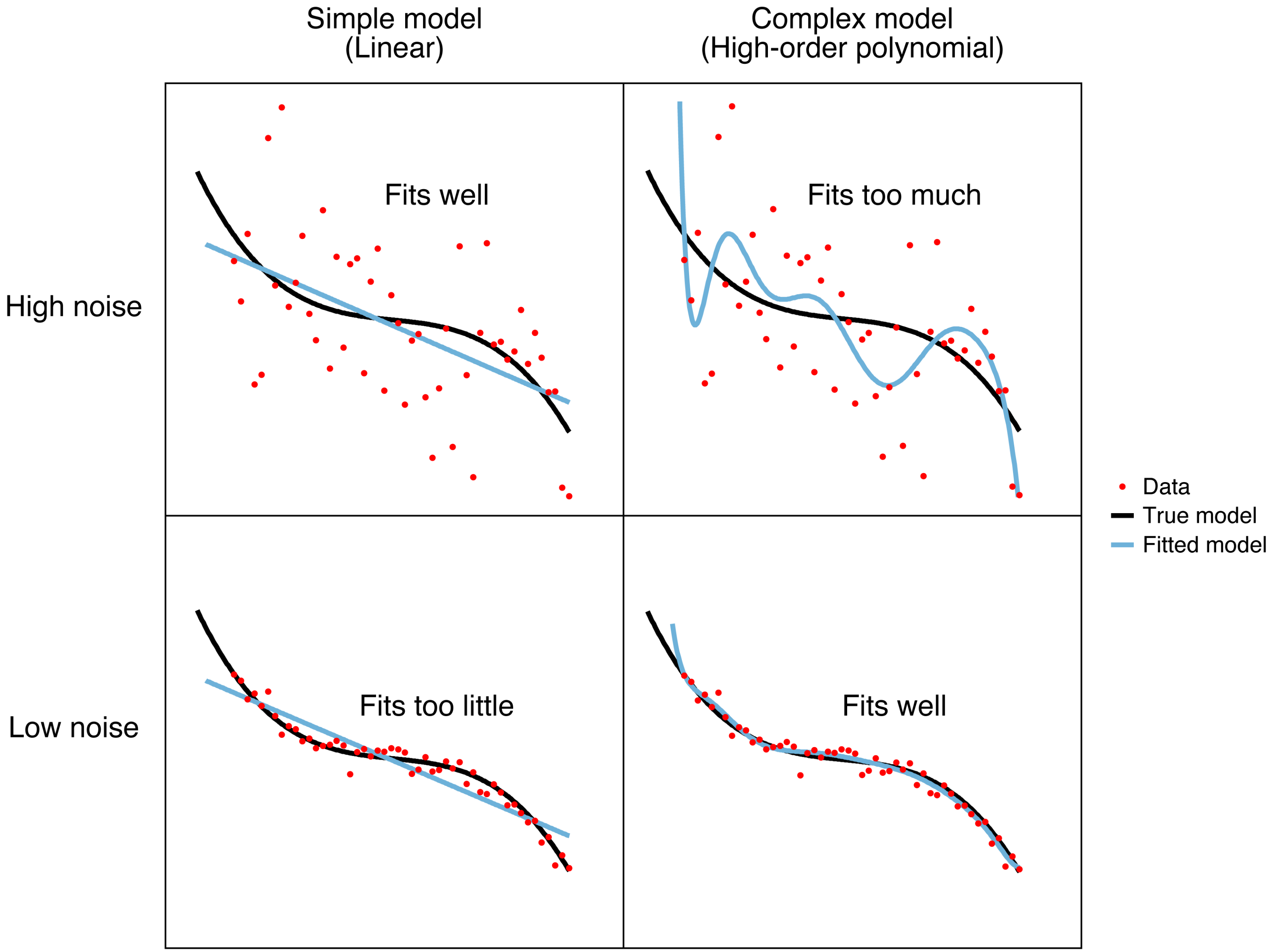
# Simple models vs. complex models



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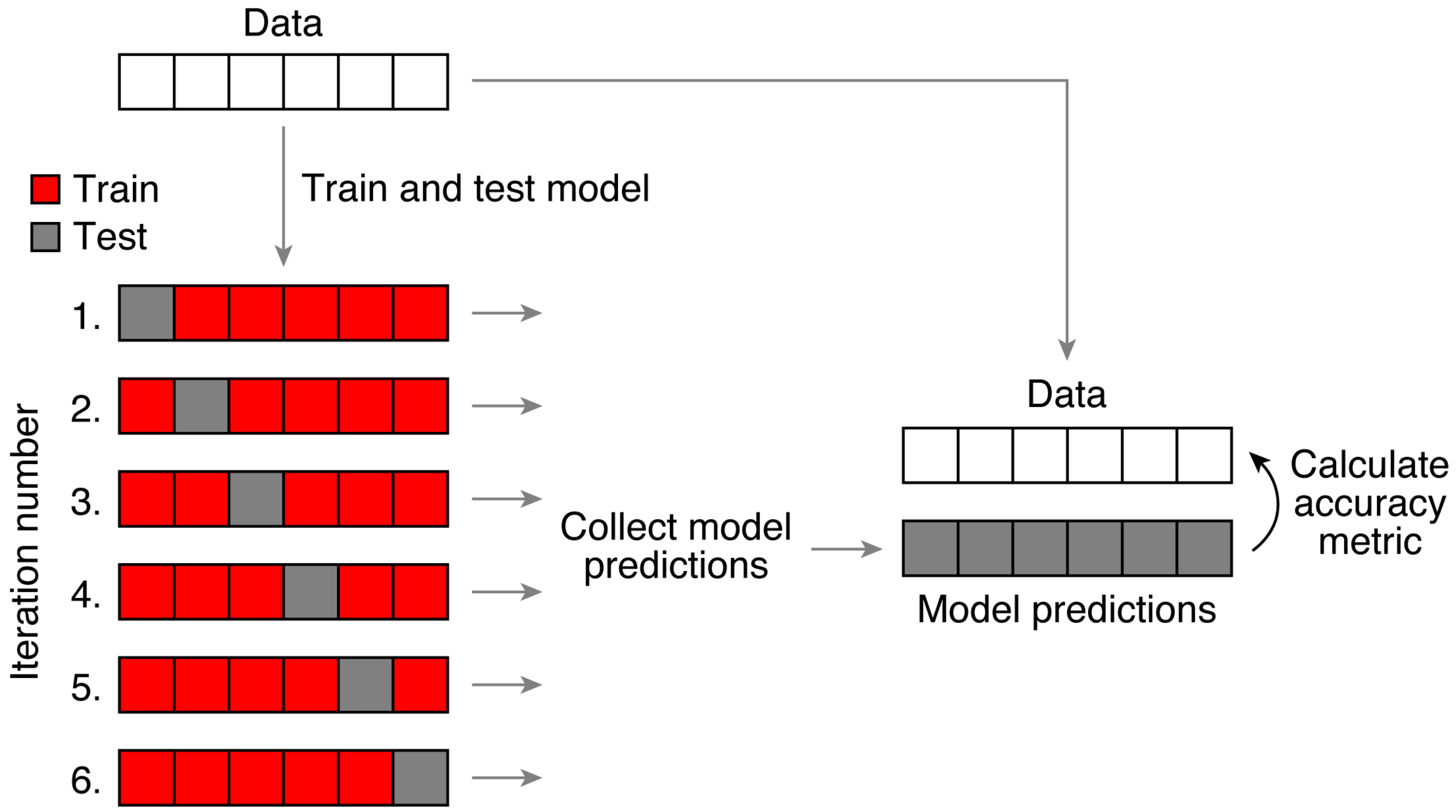
## Cross-validation

- Goal: estimate true accuracy of a model
- Approach:

  Leave some data out<br>
  Fit model<br>
  Evaluate model on left-out data

# Leave-one-out cross-validation



Data

🟥 Train

■ Test

Train and test model

Iteration number

1.

2.

3.

4.

5.

6.

Collect model predictions

Data

Model predictions

Calculate accuracy metric

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